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A DIGITAL COMPUTER PROGRAM
FOR THE GEOMETRICALLY NONLINEAR ANALYSIS OF
AXISYMMETRICALLY LOADED THIN SHELLS OF REVOLUTION

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I. INTRODUCTION TO THE DIGITAL COMPUTER PROGRAM

A. PROGRAM CAPABILITIES

A digital computer program has been developed for the geometrically nonlinear analysis of thin shells of revolution subjected to axially symmetric loads and thermal gradients. The thickness of the shell wall, the modulus of elasticity of the wall material, the load, and the thermal gradient may be smooth functions of the meridional arc length. The boundaries of the shell may be closed, free, or fixed.

The computer program is based upon a proportional loading concept and automatically solves the nonlinear equations for the displacements, rotations, internal forces, bending moments and stresses at an arbitrary number of locations along the meridian at each load step. The solution routine continues until the shell has reached a position of unstable equilibrium or until a specific number of load steps have been taken.

B. BASIC EQUATIONS

The geometrically nonlinear analysis used in this program is based upon the Kirchhoff hypothesis and the assumption of small finite angle changes. According to Reissner (Ref. 1), the governing equations for axisymmetrically loaded shells of revolution can be given in the form*

$$\left(\frac{r_o D}{\alpha_o}\right) \beta'' + \left(\frac{r_o D}{\alpha_o}\right)' \beta' - \left[\frac{D r_o'^2}{r_o \alpha_o} + \sigma \left(\frac{r_o' D}{\alpha_o}\right) \right] \beta + \alpha_o (r_o H) \sin \phi_o = \Gamma_1 \quad (1)$$

$$\left(\frac{r_o}{c \alpha_o}\right) (r_o H)'' + \left(\frac{r_o}{c \alpha_o}\right)' (r_o H)' - \left[\frac{r_o'^2}{c r_o \alpha_o} + \sigma \left(\frac{r_o'}{c \alpha_o}\right)' \right] (r_o H) - \alpha_o \beta \sin \phi_o = \Gamma_2 \quad (2)$$

* The notation used here is slightly different from Reissner's. Further, his equations contained no temperature terms.

where

$$\Gamma_1 = \left[\frac{3}{2} \frac{r'_0 z'_0 D}{r_0 \alpha_0} - \frac{\sigma}{2} \left(\frac{z'_0 D}{\alpha_0} \right)' \right] \beta^2 + \alpha_0 r_0 V \cos \phi_0 + \alpha_0 \beta \left[(r_0 H) \cos \phi_0 + r_0 V \sin \phi_0 \right]$$

$$\begin{aligned} \Gamma_2 = & \left[2 \frac{z'_0 r'_0}{C r_0 \alpha_0} + \sigma \left(\frac{z'_0}{C \alpha_0} \right)' \right] (r_0 H) \beta + \sigma \frac{z'_0 \beta' (r_0 H)}{\alpha_0 C} - \frac{1}{2} \alpha_0 \beta^2 \cos \phi_0 + \left[\frac{r'_0 z'_0}{C r_0 \alpha_0} + \sigma \left(\frac{z'_0}{C \alpha_0} \right)' \right] r_0 V \\ & + \sigma \frac{z'_0}{C \alpha_0} (r_0 V)' + \left[\frac{z_0'^2 - r_0'^2}{C r_0 \alpha_0} - \sigma \left(\frac{r'_0}{C \alpha_0} \right)' \right] r_0 V \beta - \sigma \frac{r'_0}{C \alpha_0} \beta' r_0 V - \sigma \frac{r'_0}{C \alpha_0} \beta (r_0 V)' \\ & - \frac{(r_0^2 P_n)'}{C} - \left[\sigma \left(\frac{r'_0}{C r_0} + \beta \frac{z'_0}{C r_0} \right) - \frac{C'}{C^2} \right] (r_0^2 P_n) - C (AT)' \end{aligned}$$

and where a prime denotes differentiation with respect to ξ , the meridional coordinate, and the subscript 0 refers to the undeformed shell. The change in the tangent angle to the meridian β is defined as

$$\beta = \phi_0 - \phi$$

The internal stress resultants are given by

$$N_\xi = H \cos \phi_0 + V \sin \phi_0 + \beta [H \sin \phi_0 - V \cos \phi_0] \quad M_\xi = D \left[\frac{\beta'}{\alpha_0} + \sigma \left(\beta \frac{\cos \phi_0}{r_0} + \frac{1}{2} \beta^2 \frac{\sin \phi_0}{r_0} \right) \right]$$

$$Q = -H \sin \phi_0 + V \cos \phi_0 + \beta [H \cos \phi_0 + V \sin \phi_0] \quad M_\theta = D \left[\beta \frac{\cos \phi_0}{r_0} + \frac{1}{2} \beta^2 \frac{\sin \phi_0}{r_0} + \sigma \frac{\beta'}{\alpha_0} \right]$$

$$\alpha_0 N_\theta = (r_0 H)' + r_0 \alpha_0 P_n$$

where

$$r_0 V = \int r_0 P_v \alpha_0 d\xi$$

and the displacements can be calculated using

$$u = \frac{r_0}{C} (N_\theta - \sigma N_\xi) + r_0 A T$$

$$w = \int \left[\alpha_0 (\sin \phi - \sin \phi_0) + \frac{\alpha_0}{C} (N_\xi - \sigma N_\theta + C A T) \sin \phi \right] d\xi$$

All of the physical quantities are defined in accordance with Figures 1 and 2, and σ is Poisson's ratio, A is the temperature coefficient of expansion, T is the temperature above that of zero stress and strain, and

$$C = E h$$

$$D = E h^3 / 12 (1 - \sigma^2)$$

where E is Young's modulus. When the shell wall is of thin skin sandwich construction

$$h = \sqrt{3} t_h$$

$$E = 2 t_s E_s / (\sqrt{3} t_h)$$

where t_s is the skin thickness, t_h is the distance between skins and E_s is the Young's modulus of the skins.

For this analysis α_0 has been taken as unity. For a more detailed discussion on this point, refer to Appendix A of (Ref. 2).

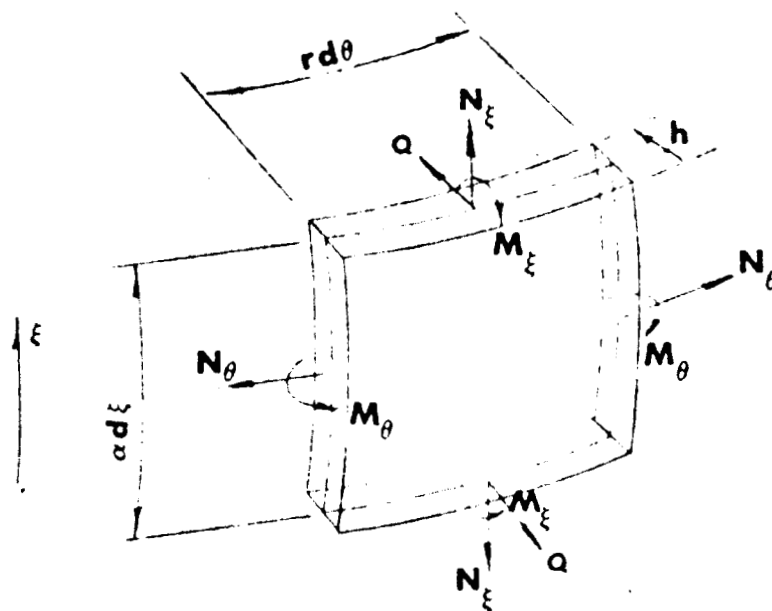


FIGURE 1

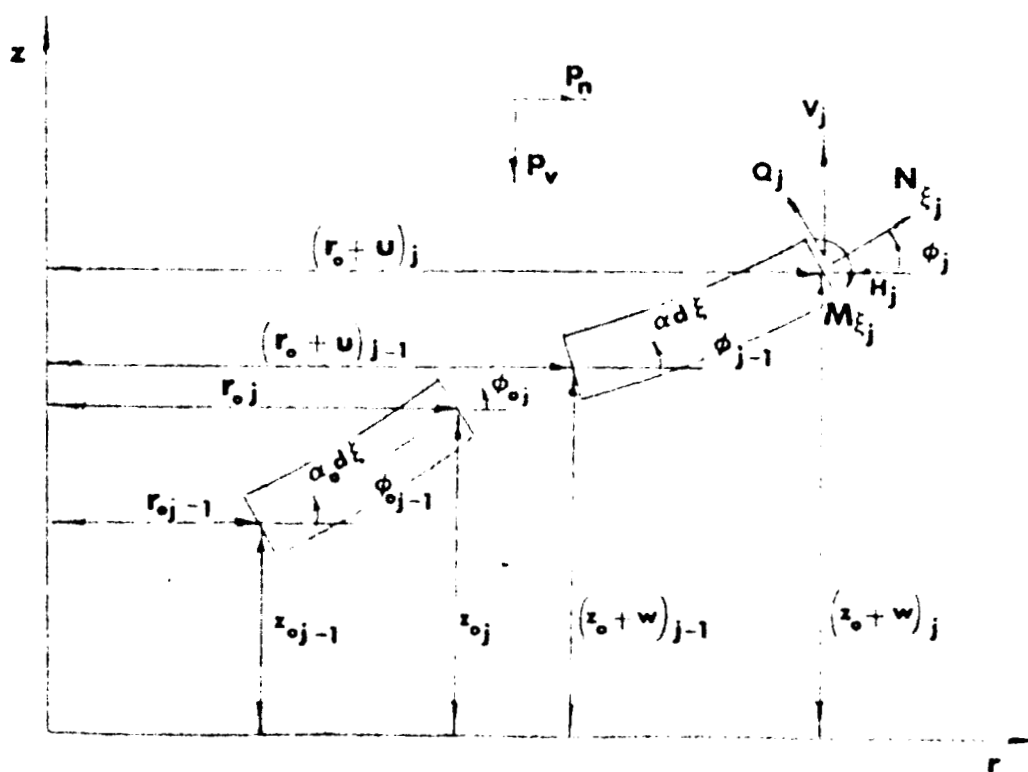


FIGURE 2

II. THE NUMERICAL ANALYSIS

The nonlinear thin shell equations, Eqs. 1, 2, are expanded in terms of $\tilde{\beta}$ and \tilde{H} (which are caused by the loads \tilde{V} and \tilde{p}_n) and $\delta\beta$ and δH (which are attributed to small changes in the loads δV and δp_n). It is then assumed that $\tilde{\beta}$ and \tilde{H} are known and that $\delta\beta$ and δH are unknown. Next, the expanded differential equations are written in finite difference form, with terms which are nonlinear in $\delta\beta$ and δH being treated as pseudo-loads. The resulting "linear" difference equations are repeatedly solved by a technique which is virtually identical to that used in the Arthur D. Little Thin Shell Program (Ref. 2). The i^{th} iterative solution for $\delta\beta$ and δH is computed using values for the pseudo-loads based on the $(i^{\text{th}} - 1)$ solution for $\delta\beta$ and δH . The convergence criterion for this process is that at every mesh point of the finite difference grid the inequalities

$$\left| \delta\beta_i - \delta\beta_{i-1} \right| \leq \epsilon \cdot \max \left| \delta\beta_{i-1} \right|$$

and

$$\left| \delta H_i - \delta H_{i-1} \right| \leq \epsilon \cdot \max \left| \delta H_{i-1} \right|$$

must be satisfied, where the subscript refers to the iteration number and \max

$\left| \delta\beta_{i-1} \right|$ is the largest value of $\left| \delta\beta \right|$ in the shell at the $(i^{\text{th}} - 1)$ iteration, etc.

The concept of proportional loading is used to gradually step the loads \tilde{V} and \tilde{p}_n up to those values which cause buckling. Initially, the incremental loads δV and δp_n are chosen by the program user to be approximately 10 to 20 percent of the expected buckling load. These loads are applied to the

shell and a nonlinear solution is obtained for $\delta\beta$ and δH . The applied loads are then increased by the original increments and a new solution is obtained for $\delta\beta$ and δH . This process repeats so that at any stage of loading \tilde{V} and \tilde{p}_n are integer multiples of δV and δp_n and $\tilde{\beta}$ and \tilde{H} are the sum of the $\delta\beta$'s and δH 's respectively. If at one load step the iteration processes for the solution do not converge after a specified number of iterations, buckling is assumed to have occurred. The loads are then reduced to the last values which did not cause buckling, δV and δp_n are automatically reduced by changing a load multiplier by a factor of 10, and the load is advanced by the smaller increments until nonconvergence occurs again. This load reduction process is then repeated several times to improve the precision of the buckling load. Finally, a complete record of displacements, rotations and internal forces, bending moments and stresses is printed at each step of the loading process.

The parameters involved in the load stepping and iteration procedures have been fixed at values which were found to produce satisfactory results in applications to thin shallow shells for which previous results were available. It is expected that they will be satisfactory for most other applications. The parameters are assigned values at the beginning of the program as follows:

NUMIT	maximum number of iterations allowed	15
NSTEP	maximum number of steps taken during each step-size cycle	15
NCY	number of step-size reduction cycles	3
EPS	convergence criterion	.001

III. PROGRAM DESCRIPTION

The basic concept of the program is the same as that of the Arthur D. Little program (Ref. 2). That is, the main program contains the numerical analysis and the input-output routines, while the description of the shell and its loading are contained in a group of FUNCTION subprograms as described in Section IV.

A logical flow diagram of the main program is presented in the Appendix.

IV. INSTRUCTIONS FOR PROGRAM OPERATION

A. SUBROUTINE PREPARATION

Every problem to be run requires nine FUNCTION-type subprograms which describe the loads and provide the basic program with the physical characteristics of the shell as a function of meridional arc length, A :

1. $R(A)$: radial distance (inches)
2. $Z(A)$: axial distance (inches), $Z(0) = 0$.
3. $T(A)$: thickness (inches) = t for solid walls
= $\sqrt{3} t_h$ for thin skin sandwich construction*
4. $E(A)$: elastic modulus (psi) = E for solid walls
= $\frac{2 t_s}{\sqrt{3} t_h} E_s$ for thin skin sandwich construction*
5. $CS1(A)$: dimensionless constant used in stress calculations = 1 for solid shell portions
= $E_s/E(A)$ for sandwich portions
6. $CS2(A)$: dimensionless constant used in stress calculations = 6 for solid shell portions
= $2\sqrt{3} = 3.464102$ for sandwich portions
7. $AT(A)$: net thermal strain, AT (inches per inch) where A is the coefficient of linear thermal expansion and T is the net temperature change
8. $VD(A)$: incremental vertical force (lb/in) (should be 10 to 20 percent of expected buckling load.)**

* See page 3, Section I, for further explanation.

** In treating the boundary conditions for a closed initial end, either V or z' must tend to zero as $r \rightarrow 0$ in order to preserve a finite value for fractions of the form Vz'/r . This program arbitrarily requires $VD(0) = 0$ if $IS1 = 0$, which means that there can be no concentrated load at $r = 0$. The same restriction exists in the Arthur D. Little program (Ref. 2) although it is not made explicit in the program description.

9. PD(A) : incremental radial component of pressure (lb/in²)
(should be 10 to 20 percent of expected buckling load).

The FUNCTIONS R, Z, T, AT, VD, and PD are typified by examples given in the report on the Arthur D. Little Thin Shell Program (Ref. 2). A FUNCTION typical of E, CS1, or CS2 for a solid shell would be:*

```
FUNCTION CS2(A)
A = A
CS2 = 6
RETURN
END
```

For a shell which is solid for $0 \leq A < 37.2$ and sandwich for $A \geq 37.2$ the same FUNCTION would be:

```
FUNCTION CS2(A)
IF (A - 37.2) 10, 20, 20
10    CS2 = 6
    RETURN
20    CS2 = 3.464102
    RETURN
END
```

B. INPUT PREPARATION

Each program to be run also requires 2 data cards. Their FORMAT and content are as follows:

Card 1: FORMAT (12A6)

The first 72 columns are used as a title for the program printout and can contain any alphabetic or numeric comments desired by the user.

* Note the dummy statement $A = A$ is necessary since the FORTRAN language requires that the argument of the FUNCTION be used.

Card 2: FORMAT (2I4, 2F 10.4)

LIST N, IPO, S, P, IS1, IS3

N is the total number of mesh points, i.e., one more than the number of mesh intervals, $N \leq 401$

IPO is the output printing frequency, so that all mesh points need not be printed out. The program will always print out both ends, regardless of the value of IPO

S is the total arc length of the shell in inches

P is Poisson's ratio

IS1 specifies the initial boundary condition

IS3 specifies the terminal boundary condition

<u>Value of IS1 or IS3</u>	<u>Boundary Condition</u>			
0	closed	β	=	$u = 0$
1	free	H	=	$M = 0$
2	fixed	β	=	$u = 0$

Note that since this program is limited to homogeneous edge conditions it is not necessary to read in boundary values of the pertinent variables.

Note that decimal points and/or exponents using the E notation on the card will override the F10.4 specification.

C. PROGRAM DECK

The program deck should conform to the FORTRAN MONITOR SYSTEM and the following Jet Propulsion Laboratory operations procedures:

1. Identification card (as specified by Jet Propulsion Laboratory)
2. XEQ card

3. Programs to be compiled
4. Binary program deck (or decks)
5. DATA card
6. Data cards (2 cards per problem)

Any number of problems can be run consecutively provided they all require the same set of subroutines. If different subroutines are required, the above card sequence must be repeated as a new file.

D. TAPE REQUIREMENTS

The following tapes are used by the program:

1. Logical tape 5 - input tape (FMS A2)
2. Logical tape 6 - output tape (FMS A3)
3. Logical tape 8 - scratch tape (FMS B1)

If the program is to be used on a system which does not conform to the above, the required program changes are trivial.

E. OPERATING PROCEDURE

The following instructions should be used together with the FORTRAN MONITOR SYSTEM:

1. Prepare subroutines and data cards and arrange as indicated in Section D above.
2. Load the card deck onto Logical tape 5 either on-line via the card reader or off-line via the 1401.
3. Ready all tapes.
4. Press START. Program will compile prior to first problem if necessary.

F. OUTPUT

At the end of a run, all output is on Logical tape 6 in the order in which it was run. The output tape is not rewound and there is no end of file mark between problems or after the last problem. The output tape should be printed off-line under program control.

V. OUTPUT FORMAT

Initially the program prints the input quantities, N , S , P , $IS1$, and $IS3$. Then, at the end of each loading step, the title card, cycle number, step number and the number of iterations required for each step are printed. If the process did not converge for the step in question the message "CONVERGENCE FAILURE---STEP SIZE REDUCED" is printed as the only output. If the process did converge, the resultant output is arranged in two parts: "A", the forces, moments, displacements, and rotations for each selected printout value of the arc length; and "B", the stresses at the inner and outer fibers of the shell. The end of a problem for which the buckling load was successfully found is indicated by the message "SHELL BUCKLED AT _____ TIMES BASIC LOAD INCREMENT." If the buckling load was not found, the message "DID NOT BUCKLE AFTER 15 STEPS" is printed at the bottom of the last page of output.

The interpretation of the buckling load in terms of multiples of the basic load increment is made clear by an example. Suppose the output messages are as follows:

CYCLE 1	STEP 1	5 ITERATIONS
(Normal "A" and "B" output)		
CYCLE 1	STEP 2	6 ITERATIONS
(Normal "A" and "B" output)		
CYCLE 1	STEP 3	7 ITERATIONS
(Normal "A" and "B" output)		
CYCLE 1	STEP 4	10 ITERATIONS
(Normal "A" and "B" output)		
CYCLE 1	STEP 5	15 ITERATIONS
CONVERGENCE FAILURE- - -STEP SIZE REDUCED		

CYCLE 2 STEP 1 4 ITERATIONS

(Normal "A" and "B" output)

CYCLE 2 STEP 2 15 ITERATIONS

CONVERGENCE FAILURE - - -STEP SIZE REDUCED

CYCLE 3 STEP 1 4 ITERATIONS

(Normal "A" and "B" output)

CYCLE 3 STEP 2 5 ITERATIONS

(Normal "A" and "B" output)

CYCLE 3 STEP 3 15 ITERATIONS

CONVERGENCE FAILURE - - -STEP SIZE REDUCED

SHELL BUCKLED AT 4.12 TIMES BASIC LOAD INCREMENT

This means that the program took 4 successful load steps of the basic size, 1 successful step 1/10th that size, and 2 successful steps 1/100th of the original step size. Thus, the buckling load is between 4.12 and 4.13 times the basic loading step.

Nomenclature for the "A" and "B" output is as follows:

<u>"A" Pages</u> <u>Column Heading</u>	<u>Units</u>	<u>Variable</u>	<u>FORTRAN</u> <u>Code</u>
ARC DISTANCE	in	ξ	DJ
VERTICAL FORCE	lb/in	V	VJ
HORIZONTAL FORCE	lb/in	H	HJ
HOOP FORCE	lb/in	N_{θ}	ENJ
AXIAL MOMENT	lb in/in	M_{ξ}	EMA
HOOP MOMENT	lb in/in	M_{θ}	FMT
HORIZONTAL DISPLACEMENT	in	u	UJ
VERTICAL DISPLACEMENT	in	w	WJ
ANGULAR ROTATION	rad.	θ	BJ

"B" pages Column Heading	Units	Variable	FORTRAN Code
ARC DISTANCE	in	ξ	DI
INNER AXIAL STRESS	psi	$S_{\xi i}$	AXI
OUTER AXIAL STRESS	psi	$S_{\xi o}$	AXO
INNER HOOP STRESS	psi	$S_{\theta i}$	HBI
OUTER HOOP STRESS	psi	$S_{\theta o}$	HBO
SHEAR STRESS	psi	S_s	SHS

The stresses are defined as

$$S_{\xi i} = \frac{C_1}{h} \left(H \cos \phi + V \sin \phi + \frac{C_2}{h} M_{\xi} \right) \quad S_{\xi o} = \frac{C_1}{h} \left(H \cos \phi + V \sin \phi - \frac{C_2}{h} M_{\xi} \right)$$

$$S_{\theta i} = \frac{C_1}{h} \left(N_{\theta} + \frac{C_2}{h} M_{\theta} \right) \quad S_{\theta o} = \frac{C_1}{h} \left(N_{\theta} - \frac{C_2}{h} M_{\theta} \right)$$

$$S_s^* = 1.5 \left(V \cos \phi - H \sin \phi \right) / h$$

where

$$C_1 = 1 \quad \text{for solid walls}$$

$$C_1 = E_s / E(A) \quad \text{for thin skin sandwich construction}$$

$$C_2 = 6 \quad \text{for solid walls}$$

$$C_2 = 2\sqrt{3} \quad \text{for thin skin sandwich construction}$$

* The maximum shear stress which occurs at the neutral axis is calculated. For thin skin sandwich construction the factor 1.5 should be replaced by $\sqrt{3}$. However, 1.5 is used for both solid and honeycomb walls even though it is slightly incorrect for the latter.

VI. SIGN CONVENTION

Refer to the differential element which is shown in Figures 1 and 2 of Section 1. Positive forces, moments, displacements, rotations and cylindrical coordinates are shown.

At $\xi = \xi_j$

Radial length, r_j positive outward

Vertical length, $z_1 = 0$. @ $\xi_1 = 0$.

z_j positive in vertical direction in which ξ initially increases

Angle between r axis and tangent to undistorted middle surface, ϕ_0 defined to make

$$\cos \phi_0 = (r_{0j+1} - r_{0j-1}) / 2 \Delta \xi$$

$$\sin \phi_0 = (z_{0j+1} - z_{0j-1}) / 2 \Delta \xi$$

Rotation during strain, $\beta = \phi_0 - \phi$

Horizontal displacement, u positive in positive r direction

Vertical displacement, w positive in positive z direction

H_j positive in positive r direction

V_j positive in positive z direction

M_{ξ_j} positive in positive β direction

It is important to note that these positive sign conventions are taken at the j^{th} point. In other words, the program prints out the results at the end of each interval. The forces and moments at the $\xi_j - 1$ position are also in the positive direction but at the beginning of the interval. These values are never printed out--except at the initial boundary.

For that case, $\xi_1 = 0$, $j = 1$

β_1	positive as before, i.e., when ϕ decreases when strained
u_1	positive in positive r direction
w_1	positive in positive z direction
H_1	positive inward
V_1	positive in negative z direction
M_{ξ_1}	positive in negative β direction.

This sign convention is the same as that used in (Ref. 2).

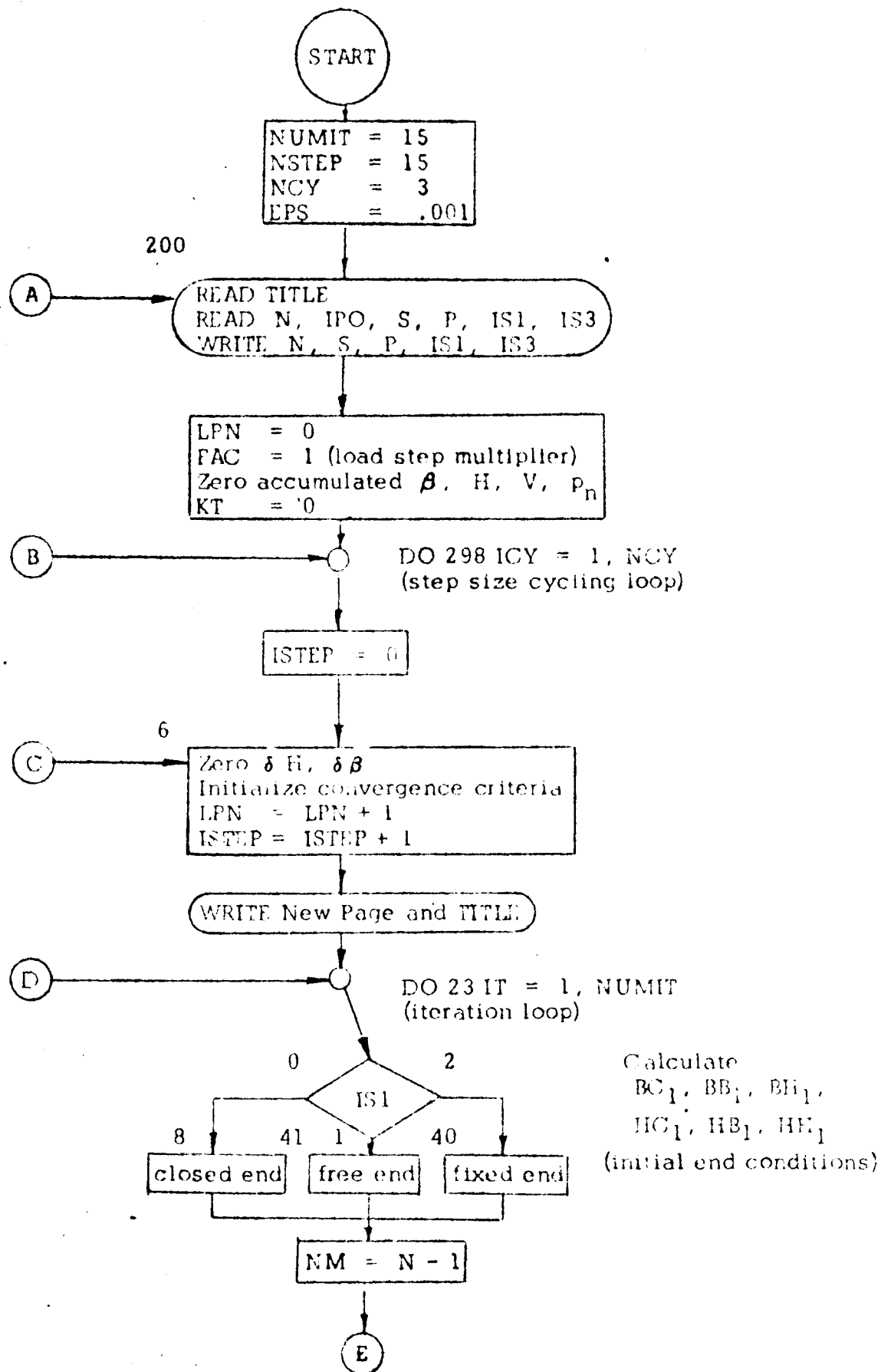
VII. REFERENCES

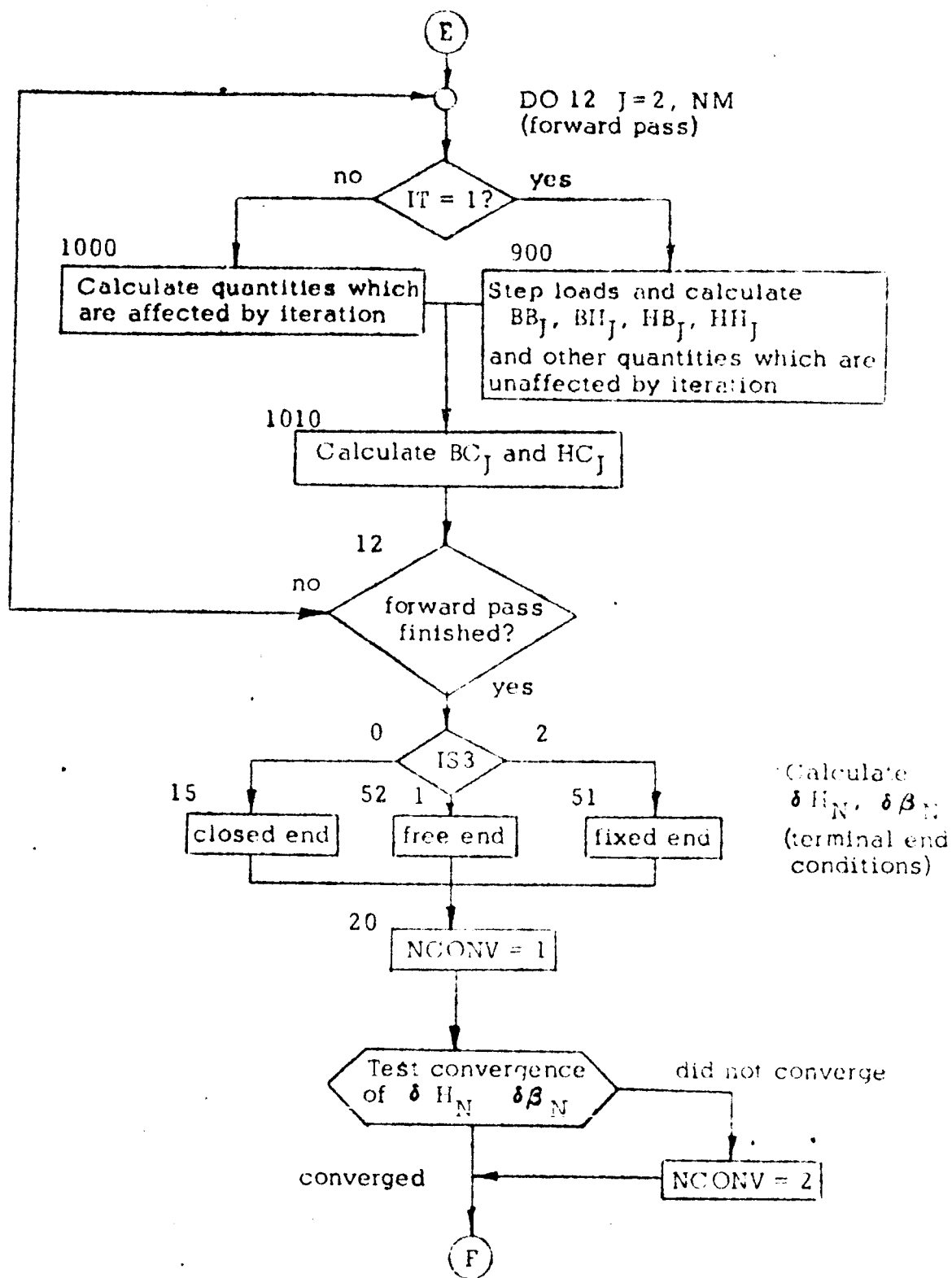
1. Reissner, E., "On Axisymmetric Deformations of Thin Shells of Revolution," Proc. Symp. in Appl. Math., Vol. 3, 1950, p. 32.
2. A. D. Little, Inc., "A Digital Computer Program for the General Axially Symmetric Thin-Shell Problem," Prepared for JPL, Jan., 1963.

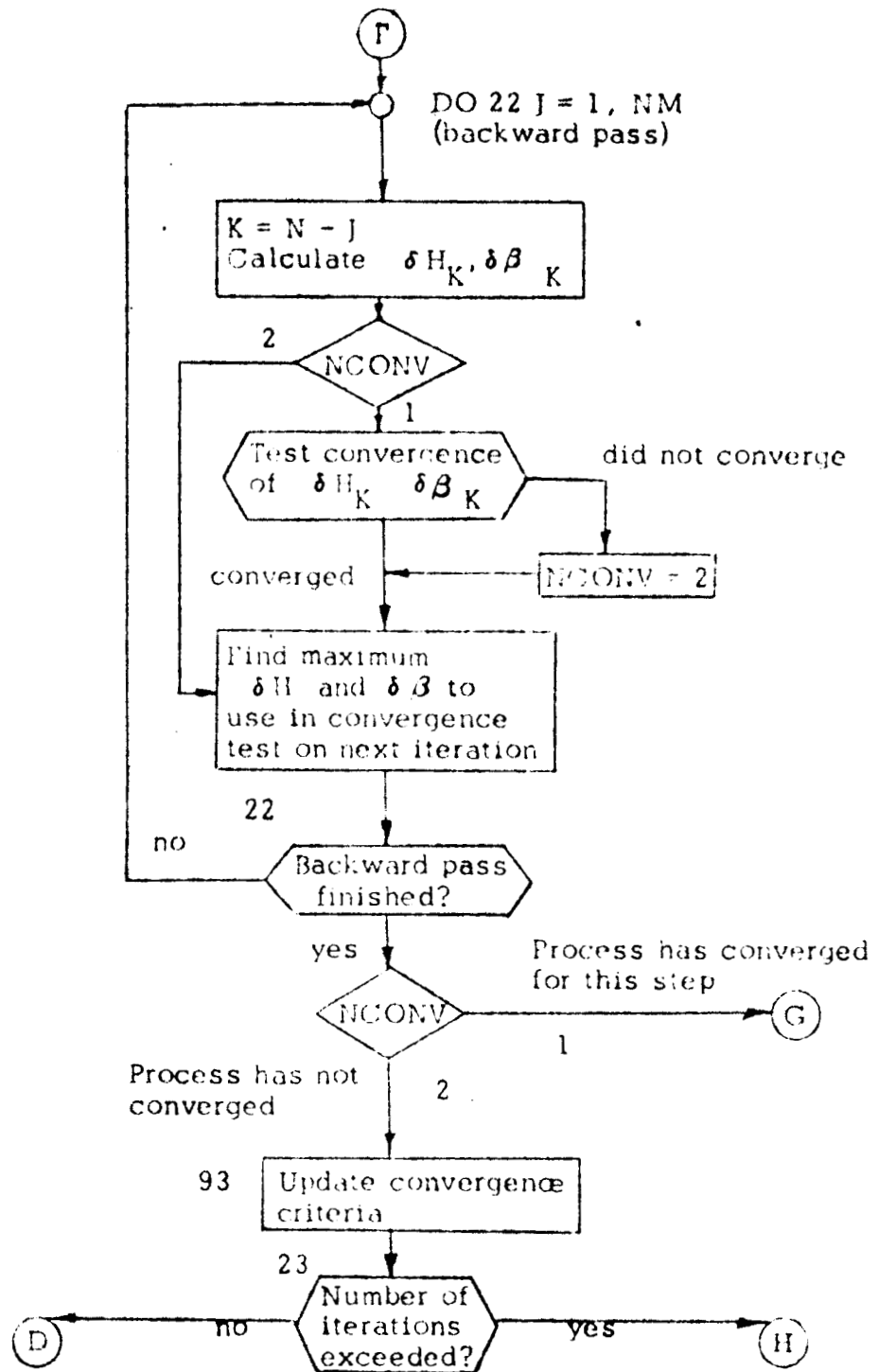
APPENDIX

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GEOMETRICALLY NONLINEAR THIN SHELL PROGRAM FLOW CHART







24

C

For all J
 $\beta_J = \beta_J + \delta \beta_J$
 $H_J = H_J + \delta H_J$

WRITE Cycle, step, and iteration numbers

REWIND Tape 8

LPO = IPO-1
 LPP = 1
 LT8 = 0

DO 26 J = 1, N
 (displacement, moment, stress loop)

Calculate
 EMA, EMT, ENJ, VJ, WJ
 giving special consideration
 to boundary conditions
 LPO = LPO+1

LPO = IPO
 ?

yes

LPO = 1

LPP = 50
 ?

no

no

yes

yes

Last point
 ?

no

LPP = 0
 LPN = LPN+1
 WRITE New Page and TITLE

WRITE moments and displacements
 Calculate stresses
 WRITE stresses on Tape 8
 LT8 = LT8+1

26

J = N
 ?

no

yes

500

READ Stresses from Tape 8
 and PRINT them

Shell has not buckled

no

yes

ISTEP
 = NSTEP
 ?

WRITE "DID NOT BUCKLE"

Take another step

Start new problem

A

